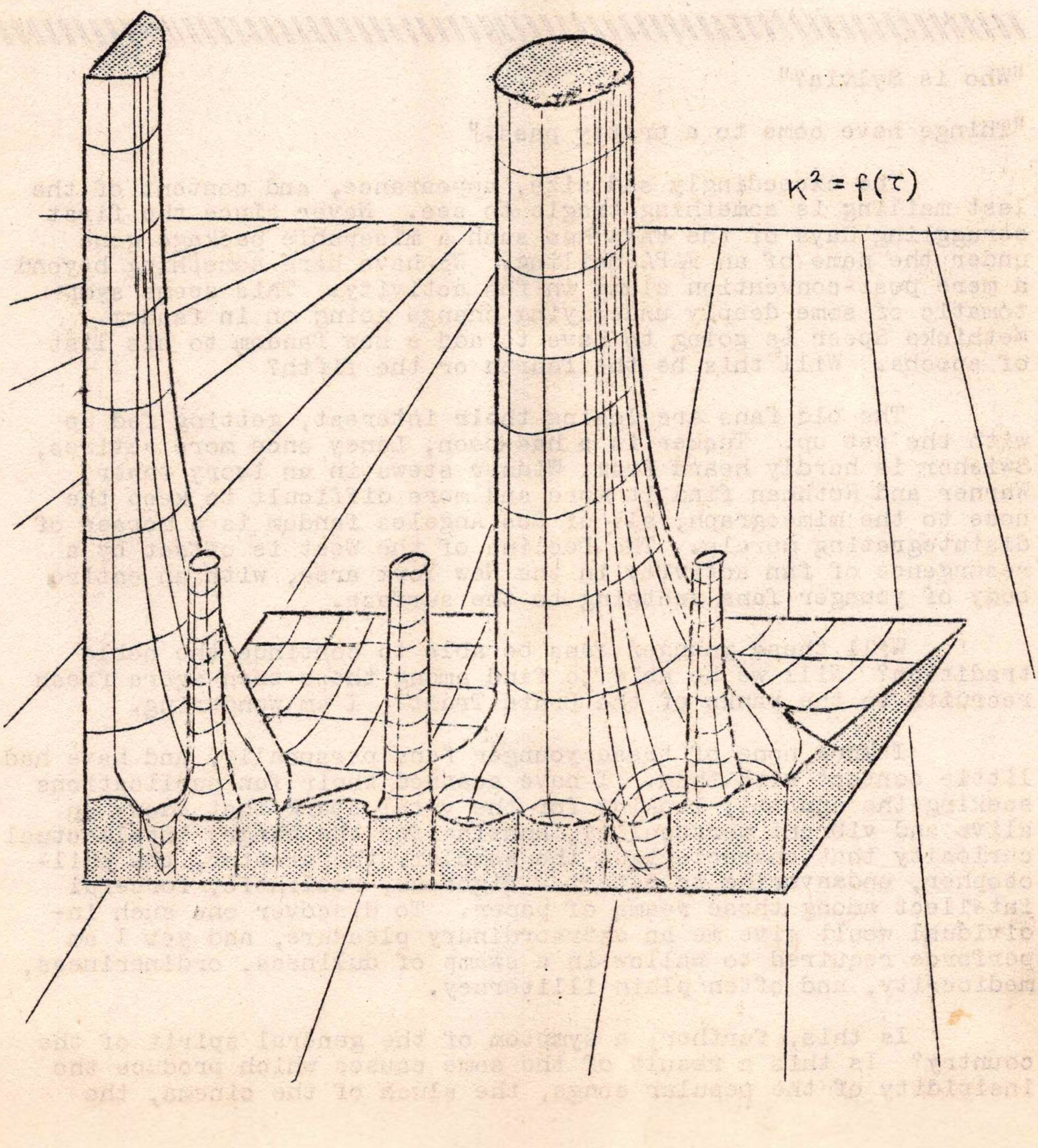
Dup

ПЛЕНУМ



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Number 5
April, 1947

For the
Fantasy Amateur
Press Association

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Philadelphia 22, Pa.

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"Who is Sylvia?"

"Things have come to a pretty pass."

The exceedingly sad size, appearance, and content of the last mailing is something tragic to see. Never since the first struggling days of the FAPA has such a miserable package gone under the name of an FAPA mailing. We have here something beyond a mere post-convention slump in fan activity. This seems symptomatic of some deeply underlying change going on in fandom. Methinks Speer is going to have to add a new fandom to his list of epochs. Will this be the fourth or the fifth?

The old fans are losing their interest, getting fed up with the set up. Tucker is a has-been, Laney once more retires, Swisher is hardly heard from, Widner stews in an ivory tower, Warner and Rothman find it more and more difficult to keep the nose to the mimeograph, all of Los Angeles fandom is a morass of disintegrating morale. The Decline of the West is offset by a resurgence of fan activity in the New York area, with an entire body of younger fans seething to the surface.

Will these younger fans be able to continue the noble tradition? Will we be able to find among these teen-agers fresh recruits to the ranks of the Brain Trust? I am wondering.

I know none of these younger fans personally, and have had little contact with them. I have scanned their fan publications seeking the unusual, looking for the vital spark that shows an alive and vibrant personality, hunting for the hungry intellectual curiosity that characterizes the scholar, the creator, the philosopher, endeavoring to detect a rigorous, realistic, forceful intellect among these reams of paper. To discover one such individual would give me an extraordinary pleasure, and yet I am perforce required to wallow in a swamp of dullness, ordinariness, mediocrity, and often plain illiteracy.

Is this, further, a symptom of the general spirit of the country? Is this a result of the same causes which produce the insipidity of the popular songs, the slush of the cinema, the dullness of modern literature, the mediocrity of government officers, the backwardness and spinelessness of our elected congress? Does this arise from the same fountain as the colorlessness of modern language, slovenliness of American enunciation, carelessness of manner, and absence of morale?

If no giants are arising in fandom, is it for the same reasons that no giants are being spawned in modern America? Where are the Bachs and the Beethovens? Where are the Shakespeares and the Byrons? Where are the Michaelangelos and the da Vincis? Where can we find another man with the stature of a Lincoln? Where is intensity and genius?

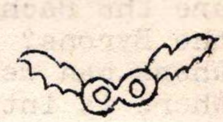
Has our civilization produced a watering-down of personality? In this day of pleasantly smiling portraits, can we duplicate the stupendous character of the Beethoven with the towering rage, the Sherman with the devastating frown?

For a few years we could point to the accomplishments of our armies and say that there we had strength and character. Yet as soon as the conflict is resolved, we are returned to a pig-trough of weakness and indecision, to a period of scampering after trivial pleasures, physical thrills, and intellectual emptiness. The insipid is exalted, the mediocre is canonized, the illiterate is idolized.

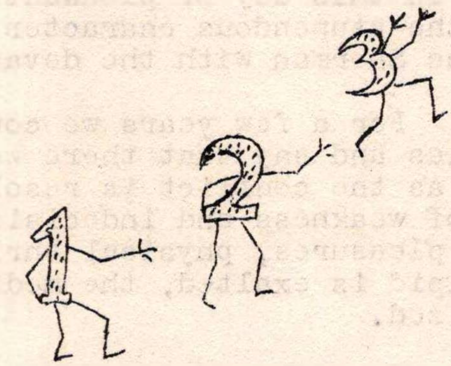
Prove me wrong, dammit. Prove me wrong.

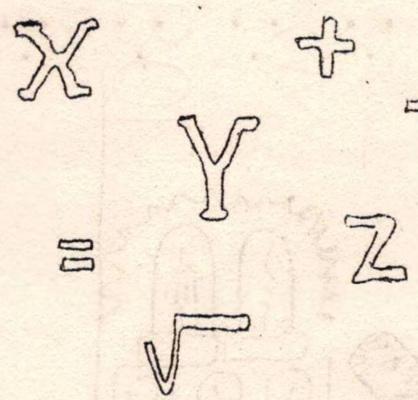
EXTENSION

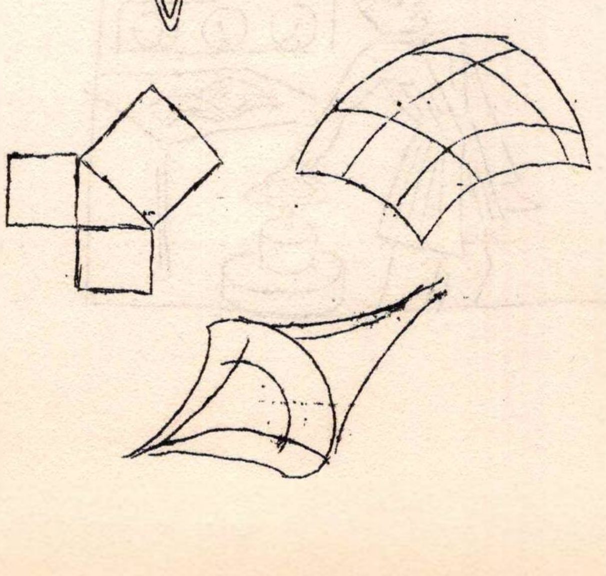
> If the party starts rough, boys, just hang on. It begins to make sense at about the 4th paragraph.



The most striking aspect of one's first encounters with the more advanced regions of mathematics and physics is the discovery of how important is the process of making extensions from particulars to generalities. Whereas in elementary geometry and physics we treated two and occasionally three dimensions of space, in the advanced courses we casually handle any number of dimensions up to infinity.







Where in elementary algebra we dealt with a few symbols at a time, we now handle an infinite number of terms at once with the convenient shorthand notation:

$$\sum_{1}^{\infty} a_n z^n$$

Or where we once handled single equations, and occasionally a pair of simultaneous equations, now we can write:

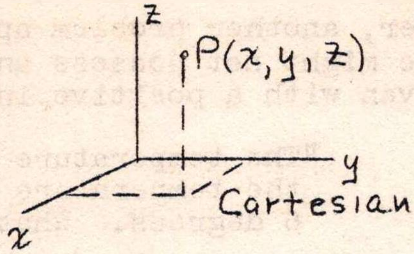
$$\bar{E} = \bar{Z}\,\bar{I}$$

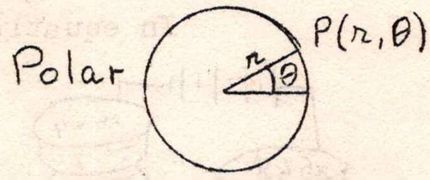
which is shorthand for the matrix equation:

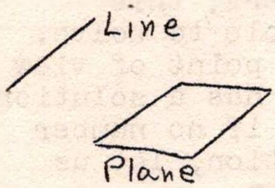
$$\begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{pmatrix} = \begin{pmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ \vdots \\ i_n \end{pmatrix}$$

which in turn is shorthand for a set of n simultaneous equations. And these can be solved with no more effort than the expenditure of a considerable quantity of arithmetic.

In first year math we learned to handle cartesian coordinates (x,y,z) and polar coordinates (r,Θ). Now we dig a little deeper, and we discover that there are no less than eleven systems of orthogonal coordinates in use -- spherical, cylindrical, parabolic, toroidal, etc. But these are not all! We find in analytical dynamics what are called "generalized coordinates" -- which are any sort of measurements which can be used to specify the position and velocities of the parts of a mechanical system.



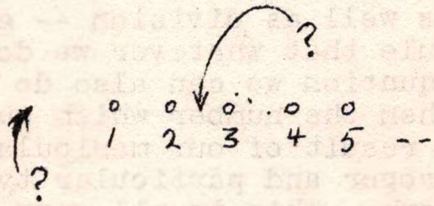




Probably the most fundamental generalization of all is the one in which the concept of number is extended from a line to a plane. So important is this that it forms the basis of all modern theory of functions.*

The idea of <u>number</u> began in antiquity with the positive integers. Just as in the first few grades of school you learn nothing more intricate than 1, 2, 3 ...., so for the first several thousand years of homo sap's development, the only items in his storehouse of numbers were the positive integers, 1, 2, 3, 4, etc.

With the advance of algebra, the need for numbers other than the positive integers became apparent, since these positive integers are only a small portion of the possible numbers, and can serve only in a limited amount of very special problems.

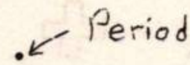


The most elementary type of algebraic problem is one like this:

"A man has x apples. He sells 5 apples, and then he has no apples left. How many apples did he have in the first place?"

We set this up in an equation like this:

$$x - 5 = 0 \; .$$



The solution makes no impossible demands upon our supply of positive integers, because it is:

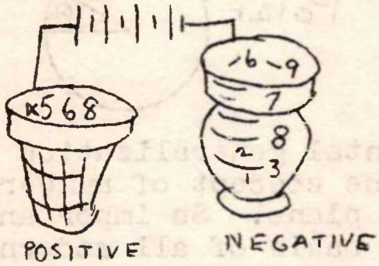
$$x = 5 \; .$$

*A function in mathematics merely is a dependency of one quantity upon another quantity. e.g., we write several equations showing how y may depend upon x: $y = 4x-3$; $y = 6x^2$; $y = \sin x$, etc. In each case we say "y is a function of x," or "$y = f(x)$."

However, another problem apparently just as simple might not possess an answer that could be given with a positive integer. Like this:

"The temperature is x degrees at noon. Later the temperature rises 10 degrees and is then 6 degrees. What was the temperature at noon?"

In equation form this reads:



$$x + 10 = 6 .$$

Now if we possessed only positive integers in our store of numbers, this equation would be impossible to solve. However, from a pragmatic point of view we know that the equation has a solution in real life, so we say: if no number exists to solve this equation, let us invent a new kind of number.

And so we do. We invent the number -4 (minus four) and we call it a <u>negative</u> integer.

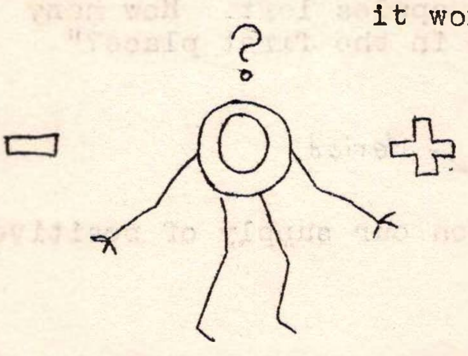
We see, then, that if we set up the equation required for a given problem, and if we apply the correct rules of multiplication and addition, as well as division -- going under the basic rule that whatever we do to one side of an equation we can also do to the other side -- then the number which automatically arises as a result of our manipulations will be of the proper and particular type that makes the answer work. This is all very pragmatic. We do not indulge in philosophical questions as to whether a negative integer is possible. All we care about is that it works.

$$a + b = c$$

$$a \cdot b = d$$

$$\frac{d}{a} = b$$

$$a + (b + c) = (a + b) + c$$



And so, when we find that a problem requires an equation such as this:

$$2x - 3 = 0,$$

we are not very shocked to find that the answer is

$$x = 3/2$$

even though this is a kind of number which belongs nowhere in the class which we called integers. For -- we have invented an entirely new type of number -- the rational fraction.

The going gets a little tougher when we come across equations of this sort:

$x^2 = 2$ -- which you get if you want to find the length of the side of a square whose area is 2. In solving it you have to invent the operation of taking a square root. (Isn't it amazing how the most trivial little problem involves the most deep concepts?) When you finish taking the square root you find that you have a number (1.4142.......) which cannot be expressed as any kind of fraction, but which you can approach as closely as you please by means of a fraction. Furthermore, if you try other equations of this sort, you will find many other numbers of this kind, and so you have invented an entirely new class of number, which is called "irrational."

Hey — how about this? —

Hah? ?

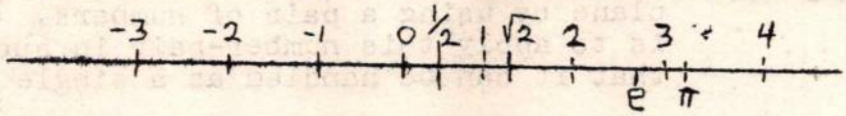
$$X^2 = -1$$

? ? ? ? ? ?

Ho?

Also belonging to the class of irrational numbers is the "transcendental number," the most familiar of which is pi (3.14159.......). Not only is it impossible to express this number as a fraction, but this number is not the solution to any algebraic equation. Yet it crops up in many apparently unrelated places.

Different as are these several kinds of numbers we have mentioned so far, they have one property in common: they can all be arranged in order of magnitude along a straight line -- like this:



For reasons which will be apparent later, all of these numbers which line up so nicely in single file are called

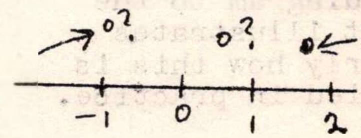
real numbers.

Up to comparatively recently this seemed to fill all requirements. All the numbers imaginable could be fitted into place along this line.

And yet there is a lack.

Something is missing.

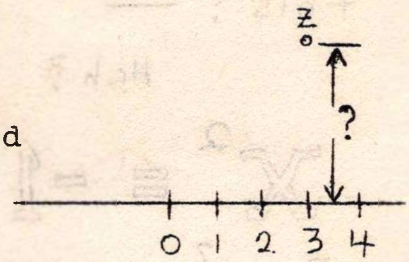
If all the imaginable numbers can be fitted into their proper niches along this thin little line -- long though it may be -- what are we to do with all this empty space going to waste on each side of the line?

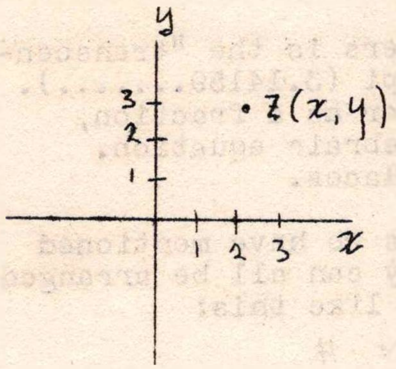


$$z^2 + 2 = 0$$

And furthermore, what are we supposed to do if we meet an equation which describes some event happening in nature, and yet which cannot be solved by any one of this infinite quantity of real numbers? We know the equation must have a solution. Yet what can it be? Such equations do come up, and wouldn't it be curious if the solutions to these equations also filled in the space along the side of our row of real numbers!

Let us begin by trying to devise some sort of number which will fill up all this empty space. In other words, let us search for a sequence of numbers such that we can assign one and only one to each point in this plane. To start with we have to have a place to start counting from, and this is conveniently supplied by the line of real numbers with which we are already familiar. So we lay this line down in the middle of the plane, and somewhere to the side we place a point z. Now to this point z we must assign a number which will describe this point z accurately, and in such a way that no other point in the plane will possess that same number.

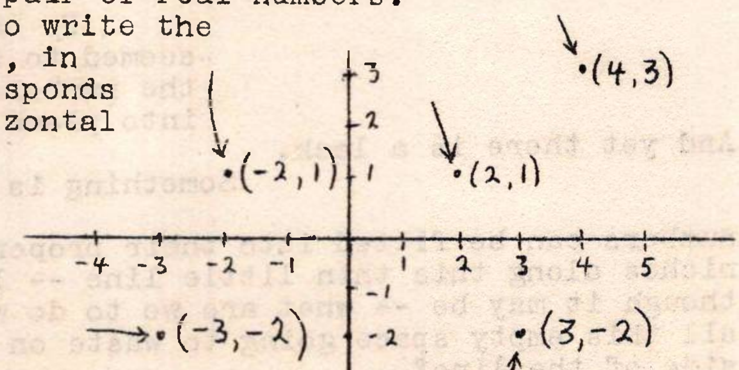




Since all of the real numbers were used up in describing all the points on the horizontal line, it is evident that a number which will describe the point z cannot be a real number. However, perhaps it is possible to describe z by means of a pair of real numbers, and then by treating this number-pair as if it were a single number. In fact, we know from the use of cartesian coordinates, that it is a simple matter to describe a point in a plane by using a pair of numbers. The trick is to apply this number-pair in such a manner that it can be handled as a single number.

If we draw a vertical line through the origin,and label each point on this line by a real number (keeping in mind, however, that we must distinguish these from the "real" real numbers, which belong solely to the horizontal line), then we see that to every point on the plane there corresponds a pair of real numbers.

We agree, as a general rule, to write the numbers in this manner: (x,y), in which the first number x corresponds to the distance along the horizontal axis, and the second number y corresponds to the distance along the vertical axis. The diagram to the right illustrates clearly how this is applied in practice.



Now if we wish to use these number-pairs in arithmetic, we must define how we are going to handle them. The definitions of the operations of addition and multiplication are purely arbitrary, and which ones we choose depends on how useful the final result becomes. The definitions which have been chosen for these number-pairs (which from now on we will call "complex numbers") turn out to produce extremely elegant and useful results.

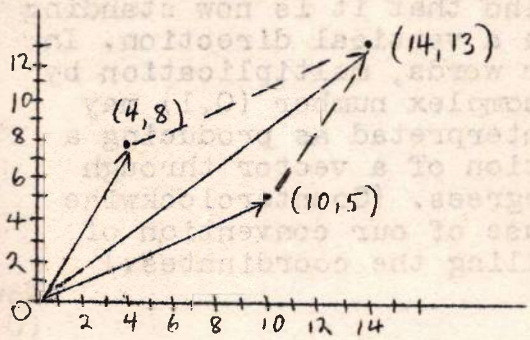
Addition we define like this:

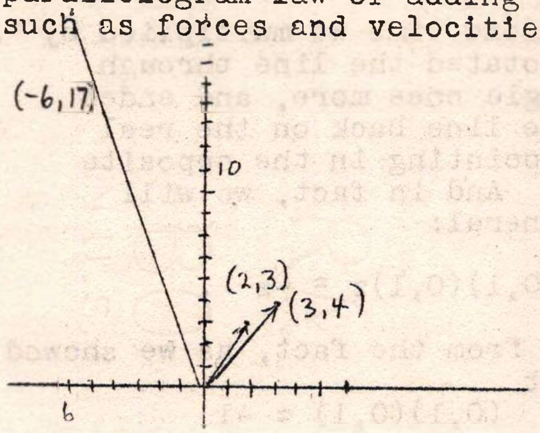
$$(a,b)+(c,d) = (a+c,\ b+d)$$

This, aside from being useful, is the most simple definition of addition that could be given. For in practice it looks like this:

$$(10,5)+(4,8) = (14,13)$$

We have simply added the two parts (or components) separately. The geometric interpretation of this is neat. Suppose we draw a line from the origin to each one of the two points corresponding to these two complex numbers, and we call these lines "vectors." Then this rule of addition has produced the ordinary parallelogram law of adding vectors such as forces and velocities:





Multiplication becomes more complicated. This is the way we are going to define multiplication:

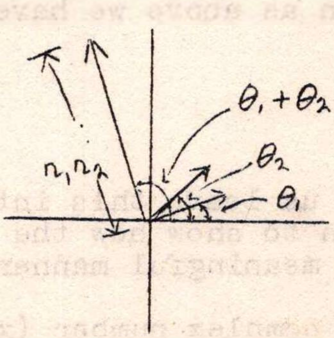
$$(a,b)(c,d) = (ac-bd,\ ad+bc).$$

So that when we perform a multiplication of complex numbers, it looks like this:

$$(3,4)(2,3) = (6-12,\ 9+8)$$
$$= (-6,17)$$

How does this appear in the geometrical interpretation?

If we plot the individual vectors, as above, we find, lo and behold, that what we have done in this multiplication, is to multiply the lengths of the vectors, and add the angles which these vectors make with the positive x axis.



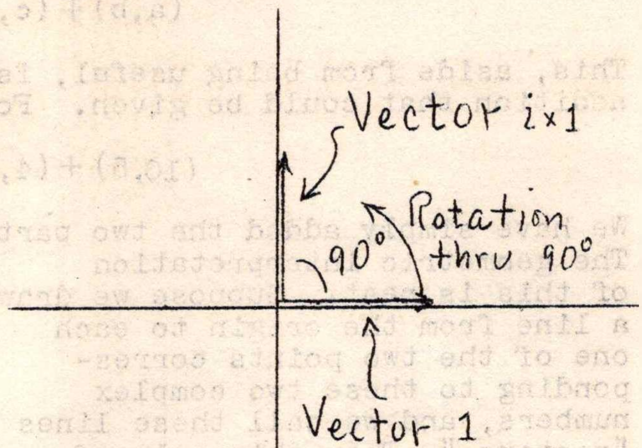
This can lead to curious results. Suppose we take the number 1, which can be written as a complex number by showing that its vertical component is zero, like this: (1,0). Now we wish to multiply it by the number (0,1). By the rule as given above, we have:

$$(1,0)(0,1) = (0,1).$$

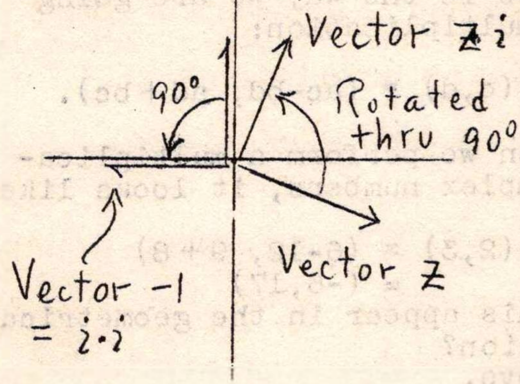
Which was to be expected, because the definition of unity is that if you multiply it by anything, you end up with the same anything.

Now suppose we multiply this again by (0,1). Like this:

$$(0,1)(0,1) = (-1,0)$$



Let us look carefully at the geometrical picture of this operation. We started with the vector (1,0), lying to the right of the origin, and identical with the real number 1. After the first multiplication we find that it is now standing up in a vertical direction. In other words, multiplication by the complex number (0,1) may be interpreted as producing a rotation of a vector through 90 degrees. (Counterclockwise because of our convention of labelling the coordinates.)



Now the second time we multiplied by (0,1) we rotated the line through a right angle once more, and ended up with the line back on the real axis, but pointing in the opposite direction. And in fact, we will find in general:

$$(0,1)(0,1)z = -z \; .$$

This comes from the fact, as we showed above, that

$$(0,1)(0,1) = -1$$

and to save ink we are going to take this curious number (0,1) and we are going to call it

"i".

Then as above we have:

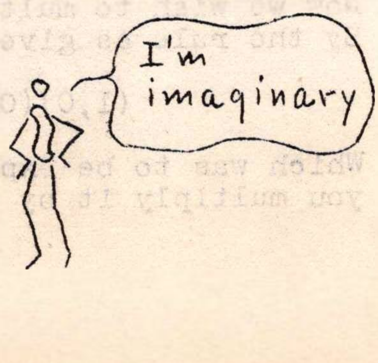
$$ii = -1$$

Or:

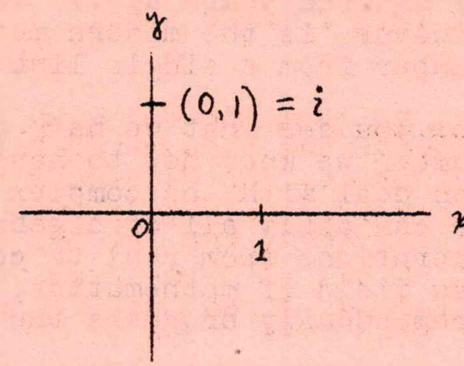
$$i^2 = -1$$

Let us leave this interesting result for a moment and make a digression to show how the number-pairs may be written in a more compact and meaningful manner.

Any complex number (x,y) has two components. As we have seen, x means the distance along the horizontal axis, and so it is called the real component. y, being the distance along the vertical axis, is called, for some obscure reason, the "imaginary" component.



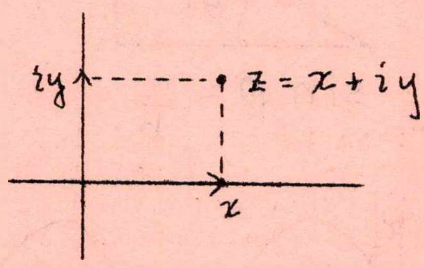
Now if a number is located right on the vertical axis, then it has no real component, and is called a pure imaginary number. So we see that the number (o,1) is located on the vertical axis, one unit above the origin, while the number (O,y) is on the vertical axis, y units above the origin. But look:



$$(0,y) = (0,1)y = iy,$$

so that

$$x + iy = (x,0) + (0,y)$$
$$= (x,y)$$



We are now enabled to dispense with the cumbersome parenthesis, and we can write any number-pair (x,y) as the sum of a real and an imaginary component

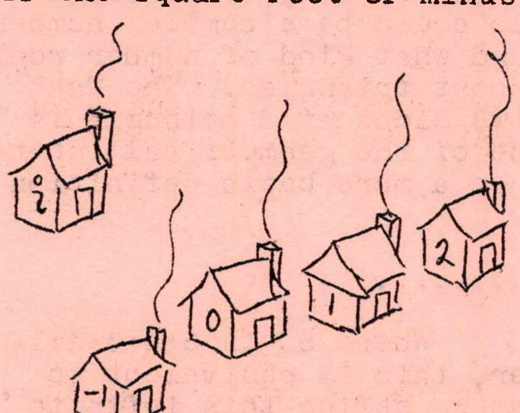
$$x + iy = z$$

in which the i in front of the y means that the y is to be added in a vertical direction.

Now going back to the expression we previously derived, $i^2 = -1$, we may now do what you have been breathlessly waiting for, and announce that i is the square root of minus 1.

$$i \equiv \sqrt{-1}$$

HUBBA-HUBBA



Now it is no use asking what number is the square root of minus 1, or asking what number when multiplied by itself will give minus 1, which is a question always asked by people who don't understand what this is all about. As we have seen, i does not fit into the set of real numbers at all. It's home is off to the side of the real numbers, and for all purposes it is not necessary to think of the square root of minus one at all.

All that is necessary and sufficient to say is this:

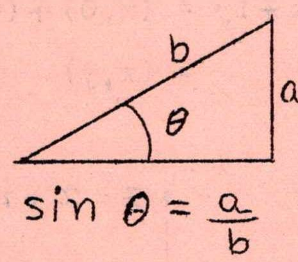
i is such a number that $i^2 = -1$.

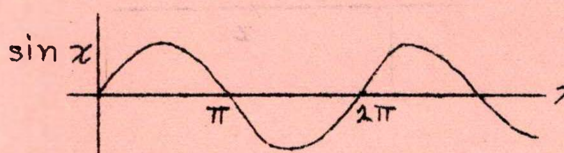
The all-important idea which makes this so useful is the notion that i is located in a direction at right angles to 1. We could, in fact, have <u>started</u> with this definition, and we could have worked backwards to end up with the rule for multiplication of number-pairs which we took as a basic definition in this paper.

That, indeed, was the historical way in which it was done (a couple of hundred years ago.) The treatment we have been using here, however, is the modern method -- an extension of the concept of number from a single line to the entire complex plane.

For you see what we have done. Whereas before we introduced complex number we knew how to handle only the real numbers x and y, now we can deal with the complex number z as if it were a single number. We can apply all of algebra and calculus to this, extending the operations from real to complex numbers, and creating an entirely new field of mathematics, Functions of a Complex Variable, which tremendously broadens the entire aspect of mathematics.

We learned in high school that the sine of an angle was the ratio of the lengths of certain sides of a triangle. We learned in calculus how to handle sines as mathematical functions -- to differentiate and integrate them -- and to recognize the curve of y = sin x. And in studying alternating currents we discovered that sine waves were quite important, and that by adding them in various ways, any kind of curve could be built up out of sine waves of various frequencies.





Now -- having complex numbers to play with -- can we extend the use of the sine to include complex numbers? In other words, can we deal with w = sin z, where z is a complex number?

A great many objections arise. Since we have always considered the sine of an <u>angle</u>, what kind of angle could be a complex number, what would be the ratio of the sides, and what kind of number would w be? We must, to begin with, forget about triangles. The fact that sin x happens to be the ratio of two sides of a triangle is really but a coincidence -- a by-product of the geometrical interpretation of the algebra. We have to use a more basic definition of the sine, and this is what it is:

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \ldots\ldots\ldots$$

Where 5! is 5·4·3·2·1.

It so happens that if z is a real number, this is equivalent to the common trigonometric sin x. So that we define this infinite series as being the sine of z for all complex values of z.

In doing this we have made a tremendous advance. We have, as a small example, made it possible to solve problems in electricity which otherwise would have been unsolvable. And all through mathematics we can make similar advances -- merely by saying that the universe of numbers covers a plane instead of a line.

An interesting sidelight on this occurs when we attempt to plot the values of sin z for all complex values of z. The sketch on the previous page shows the familiar sinusoidal nature of sin x where x is a real number. But as we have seen, this only represents a single line out of the complex plane. The drawing of sin x is, in fact, but a cross-section of the diagram of sin z. The complete picture would consist of a billowing, undulating surface, whose corrugations were even along the x axis, and which zoomed up to infinity in the y direction. For sin iy equals i sinh y, and sinh y (hyperbolic sine) does not undulate at all, but looks like the curve in which a rope will hang when two ends are held at the same level.

In mathematics there are many other functions akin to the sine and cosine functions. While the quickest way to frighten an embryo mathematician to death is to mention Bessel Functions to him, actually, they are not much more than sine functions with diminishing amplitudes. (I keep telling myself.) Then there are Legendre functions, and Elliptic Functions, and various others, each named after a certain mathematician. (Who was this guy Ellipt?) At first acquaintance they seem very abstruse, but familiarity breeds contempt, and after a while they are all just different kinds of curves on a graph, and if you have a problem which requires values of Bessel functions, you go and look them up in a table, just like you would look up sines and cosines in a table.

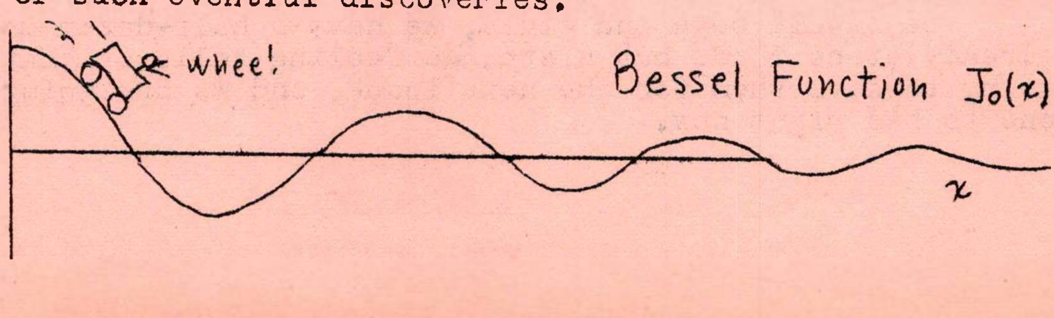
Now the ordinary chemical and engineering handbooks won't do you much good if you want something more advanced than the log of a sine. So you turn to a little book which, published by Dover at $3.50, is one of the biggest bargains in the book business. It is Jahnke & Ehmde's Tables of Functions with Formulae and Curves, written in both English and German, and full of the damndest collection of diagrams and curves you ever aaw.

The cover of this Plenum is a sample of the type of three dimensional curve which arises from plotting mathematical functions on the complex plane. This particular one is an Elliptic Modular function. (About which I know nothing -- it was just the prettiest one in the book.)

And now that we have thoroughly (oh yeah?) covered the extension of real numbers into the complex plane, we will go on from there and extend complex numbers into the complex complex cube.........

....

Editor's note: The above manuscript was found sealed in a curious container which we have not as yet been able to open. We wonder what strange and horrible fate caused the author to cease his labors on the brink of such eventful discoveries.



CODA: The article which graces the preceding pages of this sterling publication is an outgrowth of my belief that the basic concepts of mathematics are not so complex or abstruse that they cannot be understood by people without special training. While much of this material was given at the beginning of a graduate course in complex variables, I have attempted to present the matter in a way that a person with high school algebra should be able to follow.

I would appreciate seeing comments as to how clearly I have presented the subject, and whether my level of presentation was proper. Since I will be teaching Freshman physics starting this summer, I will have to get into the habit of explaining things on a level suited to the student, and it is difficult to know whether or not the students are understanding what is going on. When the operations of integral calculus have become ingrained through years of practice so that an equation acquires a semantic meaning, it requires a special effort to be understandible to people who are just learning calculus. And so this article is practice.

Though I have tried to make it as interesting as possible, there are paragraphs where it is necessary for the reader to apply a certain modicum of brain power. I hope that does not frighten too many readers away. Even such an entertaining book as "Mathematics and the Imagination" has certain passages where it is necessary for the reader to knuckle down and follow a tight line of reasoning. While some roads to mathematical understanding are easier than others, there is no completely painless path, and sooner or later one comes to a few points that have to be thought through with blood sweat and tears. .. ..

Chan Davis wins the pile of eraser scrapings for his essay on the cover of the last issue of Plenum, having applied a linear transformation of coordinates to reduce the required 20,000 words to approximately 50. Worthy though his explanation was, however, he did miss the subtle point behind it all. The cover represented <u>the plenum</u>. The Greek letter, psi, is standard notation for the Schroedinger wave function -- that "whatever it is that vibrates" which represents the waves of matter in quantum theory. The waves threading the background of the page symbolize the fullness of space. ....

Department of clarification and amplification. ..... In the last Plenum I gave the impression that "The Mislaid Charm," being published by the Prime Press, was to be lithographed. Not so. This will be a genuine, all-wool-and-a-yard-wide printed book, with twelve repeat twelve drawings by Herschel Levit, who has done the covers for several RCA-Victor albums. In this day and age how can you go wrong for $1.75? ....

Well boys and girls, we have a half-dozen more pages already stencilled, but a strange feeling tells us that we are going to save them for the next issue, and we are going to put this one to bed right now.